COMPRESSION EQUATION OF WATER-SATURATED SAND AND ITS USEFUL LIMITS

V. A. Krivtsov

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An approximate equation of state of a multicomponent medium has been introduced [1] in which the values of the compressibilities of the components occur. In this note, the actual values of the cubic content of the components are determined, an analysis of the compression equation is presented, and its useful limits are determined for water-saturated sand.

It is assumed as the basis of the model of the medium [1] that the solid component, which is a finely dispersed medium, does not form a shell, and is in the suspended sate, and that each component is compressed according to a law proper to it in the free state. This model is applicable to water-saturated sand if in a dilute (quicksand) state. Experiments conducted on water-saturated sand have demonstrated that the equations [1] are acceptable in certain cases when a skeleton [2] is present in it.

Let us determine the actual values of the cubic content of the components, analyze them and the compression equation, and estimate the degree of influence of the skeleton when the sand is compressed.

We will consider a compression equation and the cubic content of the components of water-saturated sand. Suppose that the cubic content of the gaseous, liquid, and solid components at $P = P_0$ is α_1, α_2, α_3, the specific volume of each being V_1, V_2, and V_3, the density ρ_1, ρ_2, and ρ_3 and the speed of sound in them c_1, c_2, and c_3.

At a pressure $P > P_0$ the corresponding components and parameters will be starred, i.e., $\alpha_1{}^*$, $\alpha_2{}^*$, $\alpha_3{}^*$, $V_1{}^*$, $V_2{}^*$, $V_3{}^*$, $\rho_1{}^*$, $\rho_2{}^*$, $\rho_3{}^*$, and the actual density of the three-component medium will be denoted by ρ.

Suppose the compression of each component corresponds to the Tate equation, i.e.,

$$\frac{\Delta P + B}{B} = \left(\frac{V_0}{V}\right)^{\gamma} = \left(\frac{\rho}{\rho_0}\right)^{\gamma} \tag{1}$$

where ΔP is excess pressure, and B is the mean intermolecular pressure of a component, which for air is equal to atmospheric pressure, i.e., $B_1 = \rho_1 c_1{}^2/\gamma_1 = P_0$, and for water and quartz, respectively, $B_2 = \rho_2 c_2{}^2/\gamma_2$, $B_3 = \rho_3 c_3{}^2/\gamma_3$, V_0 and V are the initial and actual values of the specific volume of a component, and γ is the isentropic exponent of a component.

The actual values of density and specific volume of the components, in correspondence with Eq. (1), are given by

$$\rho_i{}^* = \rho_i\left(\frac{\Delta P\gamma_i}{\rho_i c_i{}^2} + 1\right)^{\gamma_i^{-1}}, \qquad V_i{}^* = V_i\left(\frac{\Delta P\gamma_i}{\rho_i c_i{}^2} + 1\right)^{\gamma_i^{-1}} \quad i = 1, 2, 3 \tag{2}$$

and the cubic content of the components will depend on compressibility and on the initial cubic content of each component, i.e.,

$$\alpha_i{}^* = V_i{}^* \Big/ \sum_{j=1}^{3} V_j{}^*, \qquad i = 1, 2, 3 \tag{3}$$

Further, substituting in Eq. (3) the values of the specific volumes from Eq. (2) and dividing the numerator and denominator of the right side by the initial volume V_0 of the three component media, we obtain the actual values of the cubic content of each component

$$\alpha_i = \alpha_i\left(\frac{\Delta P\gamma_i}{\rho_i c_i{}^2} + 1\right)^{-\gamma_i^{-1}} \left\{\sum_{j=1}^{3} \alpha_j\left(\frac{\Delta P\gamma_j}{\rho_j c_j{}^2} + 1\right)^{-\gamma_j^{-1}}\right\}^{-1}, \qquad i = 1, 2, 3 \tag{4}$$

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TABLE 1

Components	Parameters			
	α	P, g/cm^2	c, m/sec	γ
Air	0.29	0.00122	340	$^4/_3$
Water	0.16	1.00	1450	7.1 [3]
Quartz	0.55	2.65	4500	3

TABLE 2

Parameter	kgf/cm^2								
	0	1.0	5.0	20	50	100	200	500	1000
ρ_1^*/ρ_1	1	1.64	3.66	9.30	18.0	30.0	50.5	102	168
ρ_2^*/ρ_2	1	1.000	1.000	1.001	1.002	1.004	1.010	1.022	1.041
ρ_3^*/ρ_3	1	1.000	1.000	1.000	1.000	1.000	1.000	1.001	1.002
α_1^*	0.29	0.20	0.101	0.042	0.023	0.0134	0.0081	0.0040	0.0025
α_2^*	0.16	0.18	0.203	0.216	0.220	0.222	0.221	0.220	0.218
α_3^*	0.55	0.62	0.696	0.742	0.757	0.765	0.771	0.776	0.779
ρ, g/cm^3	1.62	1.823	2.047	2.183	2.227	2.250	2.267	2.282	2.292

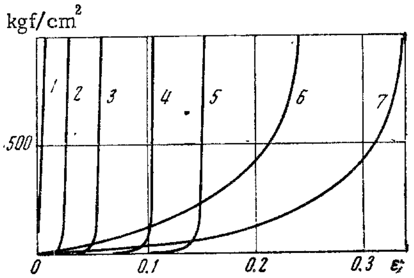

Fig. 1

Equations (2) and (4) allow us to obtain [1]

$$\rho = \sum_{i=1}^{3} \alpha_i^* \rho_i^* = \rho_0 \left[\sum_{i=1}^{3} \alpha_i \left(\frac{\Delta P \gamma_i}{\rho_i c_i^2} + 1 \right)^{-\gamma_i^{-1}} \right]^{-1}$$
$$\rho_0 = \sum_{i=1}^{3} \alpha_i \rho_i \qquad (5)$$

Introducing the deformation ε we write Eq. (5) in the form

$$\varepsilon = 1 - \frac{\rho_0}{\rho} = 1 - \sum_{i=1}^{3} \alpha_i \left(\frac{\Delta P \gamma_i}{\rho_i c_i^2} + 1 \right)^{-\gamma_i^{-1}} \qquad (6)$$

To analyze the Eqs. (4) we have obtained, and the well-known dependences (2)and (5) [2] we take the following initial data (Table 1).

The results of calculations conducted using Eqs. (2) and (4)-(6) are presented in Table 2.

It follows from Table 2 that the compressive strain values of the components in the free state differ highly for air and the two other components and are close to each other for water and qarutz. The value of $-\alpha_1^*$ decreases and that of α_3^* increases with increasing pressure; α_2^* first increases and then decreases. The physical meaning of this variation of α_2^* is that at low pressures, the volume of the gas component substatially decreases, and the gas density significantly increases, whereas the liquid and solid particles maintain their volumes and densities practically invariant. In this case, the content of the gas component in a unit volume of the medium decreases, and the content of the liquid and solid components increases. With a further increase in pressure, namely when the absolute size of the increase in α_2^* becomes greater than the increment in the gas component, α_2^* begins to decrease, and α_3^* will continue to increase, since the compressibility of the solid component is less than that of the liquid. It is not possible to obtain an analytic expression from Eq. (4) for the pressure at which α_2^* is at a maximum.

The compression Eq. (5) can be replaced down to pressures of 200 kgf/cm^2, due to the low compressibility of water and quartz in comparison with that of air, by the simpler dependence

$$\rho = \rho_0 \left\{ 1 - \alpha_1 \left[1 - \left(\frac{\Delta P \gamma_1}{\rho_1 c_1^2} + 1 \right)^{-\gamma_1^{-1}} \right] \right\}^{-1}$$

whose maximal error is less than 1%.

Let us consider the useful limits of the compression equation for water-saturated sand. To estimate the degree of influence of the skeleton on the useful limits of the equation [1] let us examine Fig. 1, which presents curves 1-5 obtained using Eq. (6) with the initial data given in Table 3, together with curves for hydrostatic compression of dense (6) and mellow (7) sand in the dry-air state taken from [4].

TABLE 3

Curves in Fig. 1	Sand components		
	α_1	α_2	α_3
1	0	0.24	0.76
2	0.02	0.22	0.76
3	0.05	0.19	0.76
4	0.10	0.14	0.76
5	0.15	0.09	0.76
6	0.24	0	0.76
7	0.34	0	0.66

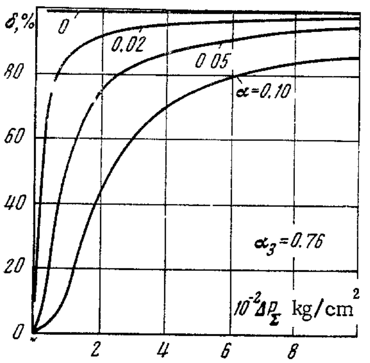

Fig. 2

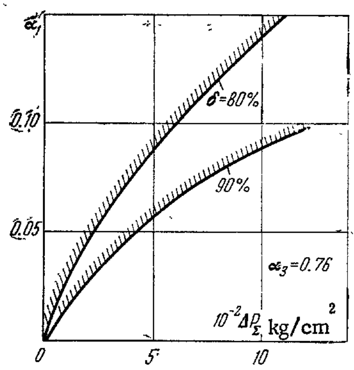

Fig. 3

A dense sand (curve 6) has a porosity of 0.24. Attached water is absent at the particle interfaces of this sand, so that capillary forces, which can be represented as the shearing strength in compression, are absent. Water-saturated sand with a skeleton has a porosity close to the porosity of condensed sand in the dry-air state and capillary forces are also absent at the particle interfaces. Consequently, compression processes for the skeletons of water-saturated sand and of dense sand in the dry-air state must be identical since they consist of the super-packing of the particles and in the condensation of the sand. Curve 7 for friable sand at pressure of 100 kgf/cm^2 attest to a similar process. It may be noted in Fig. 1 that the difference in deformations of curves 6 and 7 for fixed pressures $\Delta P \geq 100$ kgf/cm^2 remain constant and equal to 0.1.

If we assume that the compression curve for the skeleton of water-saturated sand is similar to curve 6, a comparison of curves 1-5 with this curve (Fig. 1) will allow us to assess the useful limits of Eq. (6)

Dependences are presented in Fig. 2, obtained from Fig. 1 for the relative value of pressure

$$\delta = \frac{\Delta P_l(\varepsilon)}{\Delta P_l(\varepsilon) + \Delta P_c(\varepsilon)} 100\%$$

on compressive stress at constant initial content of the solid component α_3 of 0.76 and four values of the gas component $\alpha_1 = 0$, 0.02, 0.05, and 0.10.

Here, the function $\Delta P_l(\varepsilon)$ is determined from Eq. (6) and describes curves 1-5, while the function $\Delta P_c(\varepsilon)$ describes curve 6 (Fig. 1). The values of pressure ΔP_Σ (y axis in Fig. 2) is the sum of ($\Delta P_\Sigma = \Delta P_l(\varepsilon) + \Delta P_c(\varepsilon)$).

The curves in Fig. 2 demonstrate the degree of accuracy of Eq. (6) as a function of compressive pressure and gaseous component content.

An analysis of these curves shows that when $\alpha_1 = 0$ the skeleton scarcely affects the compressive curve of water-saturated sand. With increasing α_1 the role of the skeleton in the region of low pressure substantially increases and the accuracy of Eq. (6) decreases. With further increase in compressive pressure, the influence of the skeleton on compression decreases, and the accuracy of Eq. (6) increases.

Our technique permits the useful limits of the equation [1] to be established with a given degree of accuracy if we know the compressive curves of the skeleton of sand in the dry-air state. In Fig. 3, two such limits are constructed for an actual example with accuracies of 80% and 90%. The regions under the boundary curves characterize the usefulness of Eq. (6) for different $\alpha_3 = 0.76$. In the regions above the boundary curves, Eq. (6) is not applicable at these accuracies.

For sandy soils with the same content of the components as considered above but with different remaining characteristics (form of particles, state of their surface, granulometric composition) the relationships we have obtained as experiments have shown, are satisfied to a degree sufficient for actual use.

The analysis conducted shows the usefulness of the equation [1] to a previously specified accuracy for water-saturated sand containing a skeleton. It follows from the analysis that the compression equation can be applied to a degree of accuracy sufficient for practical purposes also for other soils, in particular clay if the skeleton compression curves and their structural relations are known.

Thus the compression equation acquires a universality when applied to different multicomponent media containing a skeleton.

LITERATURE CITED

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